A NOTE ON EXTENDED BASED ON GENERALIZED HARRIS ALGORITHM USING SGN FUNCTION

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Abstract. *In this note we present new working extended algorithm which enhance the results obtained in [28]. The algorithm works well for every integers a and b, for which $a \neq 0$ and $b \neq 0$.*

Key words: Extended Euclidean algorithm, Hybrid extended algorithm, sgn function, Least absolute remainder.

Mathematics Subject Classification: 11A05, 68W01

1. Introduction

We want to solve the equation $x * a + y * b = gcd$ in respect of x and y, where $gcd \geq 1$ is a Greatest Common Divisor of given integer numbers $a \neq 0$ and $b \neq 0$. Euclidean algorithms are the aim of serious study, see [1]–[9] and [30]–[49]. Our new ideas concern computational ways and organization of these algorithms [10]–[29].

For testing purposes we will use the following computer: processor – Intel(R) Core(TM) i7-6700HQ CPU 2.60GHz, 2592 Mhz, 4 Core(s), 8 Logical Processor(s), RAM 16 GB, Microsoft Windows 10 Enterprise x64, Microsoft Visual C# 2017 x64.

2. Main Results

Using least absolute remainder and sgn function we receive the following optimized hybrid extended iterative

Algorithm 1.

```
int g = 0; x2 = 0; y1 = 0;
if (a > 0) x1 = 1; else if (a < 0) x1 = -1;
if (b > 0) y2 = 1; else if (b < 0) y2 = -1;
sng = x1 * y2; b = Math.Abs(b); a = Math.Abs(a);
if ((a & 1) == 0 && (b & 1) == 0)
```

```
do { a >>= 1; b >>= 1; g++; }
while ((a & 1) == 0 && (b & 1) == 0);
u = a; v = b;
while ((u & 1) == 0) { u >>= 1;
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else if (sng > 0) { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
else { x1 = (x1 - b) >> 1; x2 = (x2 - a) >> 1; } }
while ((v & 1) == 0) { v >>= 1;
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else if (sng > 0) { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
else { y1 = (y1 - b) >> 1; y2 = (y2 - a) >> 1; } }
if (u > v) do { q = u / v; u %= v;
if (u < 1) { x = y1; y = y2; gcd = v << g; break; }
x1 -= q * y1; x2 -= q * y2;
if ((u & 1) == 0) { do { u >>= 1;
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else if (sng > 0) { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
else { x1 = (x1 - b) >> 1; x2 = (x2 - a) >> 1; } } while ((u & 1) == 0);
if (u == 1) { x = x1; y = x2; gcd = u << g; break; } }
else { ar = v - u; if (u > ar) { u = ar; x1 = y1 - x1; x2 = y2 - x2;
do { u >>= 1;
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else if (sng > 0) { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
else { x1 = (x1 - b) >> 1; x2 = (x2 - a) >> 1; } } while ((u & 1) == 0);
if (u == 1) { x = x1; y = x2; gcd = u << g; break; } } }
q = v / u; v %= u;
if (v < 1) { x = x1; y = x2; gcd = u << g; break; }
y1 -= q * x1; y2 -= q * x2;
if ((v & 1) == 0) { do { v >>= 1;
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else if (sng > 0) { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
else { y1 = (y1 - b) >> 1; y2 = (y2 - a) >> 1; } } while ((v & 1) == 0);
if (v == 1) { x = y1; y = y2; gcd = v << g; break; } }
else { ar = u - v; if (v > ar) { v = ar; y1 = x1 - y1; y2 = x2 - y2;
do { v >>= 1;
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
```

```
else if (sng > 0) { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
else { y1 = (y1 - b) >> 1; y2 = (y2 - a) >> 1; } } while ((v & 1) == 0);
if (v == 1) { x = y1; y = y2; gcd = v << g; break; } } } } while (true);
else do { q = v / u; v %= u; if (v < 1) { x = x1; y = x2; gcd = u << g; break; }
y1 -= q * x1; y2 -= q * x2;
if ((v & 1) == 0) { do { v >>= 1;
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else if (sng > 0) { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
else { y1 = (y1 - b) >> 1; y2 = (y2 - a) >> 1; }
} while ((v & 1) == 0);
if (v == 1) { x = y1; y = y2; gcd = v << g; break; } }
else { ar = u - v; if (v > ar) { v = ar; y1 = x1 - y1; y2 = x2 - y2;
do { v >>= 1; if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else if (sng > 0) { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
else { y1 = (y1 - b) >> 1; y2 = (y2 - a) >> 1; } } while ((v & 1) == 0);
if (v == 1) { x = y1; y = y2; gcd = v << g; break; } } }
q = u / v; u %= v;
if (u < 1) { x = y1; y = y2; gcd = v << g; break; }
x1 -= q * y1; x2 -= q * y2;
if ((u & 1) == 0) { do { u >>= 1;
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else if (sng > 0) { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
else { x1 = (x1 - b) >> 1; x2 = (x2 - a) >> 1; } } while ((u & 1) == 0);
if (u == 1) { x = x1; y = x2; gcd = u << g; break; } }
else { ar = v - u; if (u > ar) { u = ar; x1 = y1 - x1; x2 = y2 - x2;
do { u >>= 1; if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else if (sng > 0) { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
else { x1 = (x1 - b) >> 1; x2 = (x2 - a) >> 1; } } while ((u & 1) == 0);
if (u == 1) { x = x1; y = x2; gcd = u << g; break; } } } } while (true);
```

and its recursive implementation as

Algorithm 2.

```
static long Euclid(long u, long v, long a, long b,
ref long x, ref long y, long x1, long x2, long y1, long y2, int g, long sng)
{ long q, ar;
if (u > v) { q = u / v; u %= v;
```

```
if (u < 1) { x = y1; y = y2; return v << g; }
x1 -= q * y1; x2 -= q * y2;
if ((u & 1) == 0) { if (u == 1) { x = x1; y = x2; return u << g; }
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else if (sng > 0) { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
else { x1 = (x1 - b) >> 1; x2 = (x2 - a) >> 1; }
return Euclid(u >> 1, v, a, b, ref x, ref y, x1, x2, y1, y2, g, sng); }
else { if (u == 1) { x = x1; y = x2; return u << g; }
ar = v - u; if (u > ar) { u = ar; x1 = y1 - x1; x2 = y2 - x2;
if ((u & 1) == 0) { if (u == 1) { x = x1; y = x2; return u << g; }
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else if (sng > 0) { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
else { x1 = (x1 - b) >> 1; x2 = (x2 - a) >> 1; }
return Euclid(u >> 1, v, a, b, ref x, ref y, x1, x2, y1, y2, g, sng); } } } }
else { q = v / u; v %= u; if (v < 1) { x = x1; y = x2; return u << g; }
y1 -= q * x1; y2 -= q * x2;
if ((v & 1) == 0) { if (v == 1) { x = y1; y = y2; return v << g; }
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else if (sng > 0) { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
else { y1 = (y1 - b) >> 1; y2 = (y2 - a) >> 1; }
return Euclid(u, v >> 1, a, b, ref x, ref y, x1, x2, y1, y2, g, sng); }
else { if (v == 1) { x = y1; y = y2; return v << g; }
ar = u - v; if (v > ar) { v = ar; y1 = x1 - y1; y2 = x2 - y2;
if ((v & 1) == 0) { if (v == 1) { x = y1; y = y2; return v << g; }
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else if (sng > 0) { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
else { y1 = (y1 - b) >> 1; y2 = (y2 - a) >> 1; }
return Euclid(u, v >> 1, a, b, ref x, ref y, x1, x2, y1, y2, g, sng); } } } }
return Euclid(u, v, a, b, ref x, ref y, x1, x2, y1, y2, g, sng); }
```

The recursive function can be called by:

```
int g = 0; x2 = 0; y1 = 0;
if (a > 0) x1 = 1; else if (a < 0) x1 = -1;
if (b > 0) y2 = 1; else if (b < 0) y2 = -1;
sng = x1 * y2; b = Math.Abs(b); a = Math.Abs(a);

if ((a & 1) == 0 && (b & 1) == 0)
```

```
do { a >>= 1; b >>= 1; g++; }
while ((a & 1) == 0 && (b & 1) == 0);
u = a; v = b;
while ((u & 1) == 0) { u >>= 1;
if ((x1 & 1) == 0 && (x2 & 1) == 0) { x1 >>= 1; x2 >>= 1; }
else if (sng > 0) { x1 = (x1 + b) >> 1; x2 = (x2 - a) >> 1; }
else { x1 = (x1 - b) >> 1; x2 = (x2 - a) >> 1; } }
while ((v & 1) == 0) { v >>= 1;
if ((y1 & 1) == 0 && (y2 & 1) == 0) { y1 >>= 1; y2 >>= 1; }
else if (sng > 0) { y1 = (y1 + b) >> 1; y2 = (y2 - a) >> 1; }
else { y1 = (y1 - b) >> 1; y2 = (y2 - a) >> 1; } }
gcd = Euclid(u, v, a, b, ref x, ref y, x1, x2, y1, y2, g, sng);
```

Numerical Example.

For testing of Algorithms 1 and 2 we will use the following main function:

```
long a, b, gcd, d1 = 0, x = 0, y = 0;
long x1 = 0, x2, y1, y2 = 0, q, u, v, sng, ar;

for (int i = 1; i < 100000001; i++) { a = i; b = 200000002 - i;
//here are placed the source code of algorithm 1
//as well as calling of recursive algorithm 2
d1 += gcd; }
Console.WriteLine(d1);
```

CPU time results are:

CPU time of Algorithm 1 is: **44.236 seconds.**

CPU time of Algorithm 2 is: **69.249 seconds.**

We will compare the time results of our Algorithms 1 and 2 to the algorithms for extended Harris algorithm using SGN function in [28] which time results are **49.753 seconds** and **90.674 seconds** for both iterative and recursive algorithms respectively.

3. Conclusion

We give one possible way of optimizing the computational process of extended Harris algorithm using SGN function.

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