ON A MODIFIED ADAPTIVE HOPF–LIKE OSCILLATOR WITH A SUITABLE, USER-FIXED "*INPUT FUNCTION OF COUPLING STRENGTH (IFCS)*"

Angel Golev, Valia Arnaudova

Abstract. *In this article we demonstrate some specialized modules for investigating the dynamics of some adaptive Hopf oscillators, an integral part of a planned much more general Web–based application for scientific computing. We also study a new modified adaptive Hopf–like oscillator with a suitable, user-fixed "input function of coupling strength (IFCS)". Numerical examples, illustrating our results using CAS MATHEMATICA are given.*

Key Words: Adaptive Hopf oscillator, Modified adaptive Hopf–like oscillator with a suitable, User-fixed "*input function of coupling strength (IFCS)*"

Mathematics Subject Classification: 65L07, 34A34

In this paper we demonstrate some specialized modules for investigating the dynamics of some new modified adaptive Hopf oscillators, an integral part of a planned much more general Web–based application for scientific computing.

1. Some modifications of the adaptive Hopf oscillator

The following modifications of Hopf oscillator are known [39]

$$\begin{cases} \dfrac{dx_1}{dt} & = & (1-(|x_1|+|y_1|))sgn(x_1)+a_1y_1 \\ \dfrac{dx_2}{dt} & = & (1-(|x_2|+|y_2|))sgn(x_2)-a_2x_2 \\ \dfrac{dy_1}{dt} & = & (1-(|x_1|+|y_1|))sgn(y_1)-a_1x_1 \\ \dfrac{dy_2}{dt} & = & (1-(|x_2|+|y_2|))sgn(y_2)-a_2x_2 \end{cases} \tag{1}$$

and

$$\begin{cases} \dfrac{dx_i}{dt} & = & (1-(|x_i|+|y_i|))sgn(x_i)+a_iy_i+c(x_k-x_i) \\ & & i,k\in\{1,2\},\ \ i\neq k \\ \dfrac{dy_i}{dt} & = & (1-(|x_i|+|y_i|))sgn(y_i)-a_ix_i+c(y_k-y_i) \\ & & i,k\in\{1,2\},\ \ i\neq k \end{cases} \tag{2}$$

where c in (2) is the *coupling strength.*

Some simulations

I For given

$$a_1 = 0.2,\ \ a_2 = 0.4,\ \ c = 0.5$$

the simulations on the system (2) for $x_1(0) = 0.9$; $x_2(0) = 0.13$, $y_1(0) = 0.8$, $y_2(0) = 0.12$ are depicted on Figure 1a.

Using operator $ParametricPlot3D$[...] in CAS Mathematica for components of solution – $x_1(t), y_1(t)$ see Figure 1b.

I.1 For given

$$a_1 = 0.1,\ \ a_2 = 0.3,\ \ c = 0.9$$

the simulations on the system (2) for $x_1(0) = 0.4$; $x_2(0) = 0.3$, $y_1(0) = 0.3$, $y_2(0) = 0.2$ are depicted on Figure 2a.

Using operator $ParametricPlot3D$[...] in CAS Mathematica for components of solution – $x_1(t), y_1(t)$ see Figure 2b.

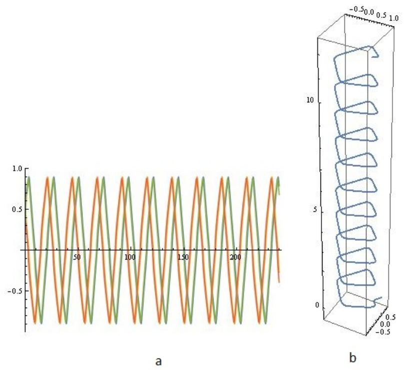


Figure 1. a) The solutions of differential system; (example I);
b) $ParametricPlot3D$[...]; (example I).

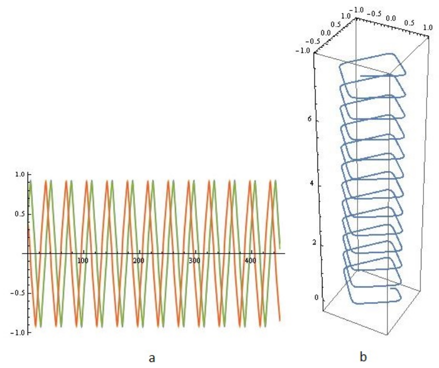


Figure 2. a) The solutions of differential system; (example I.1);
b) $ParametricPlot3D$[...]; (example I.1).

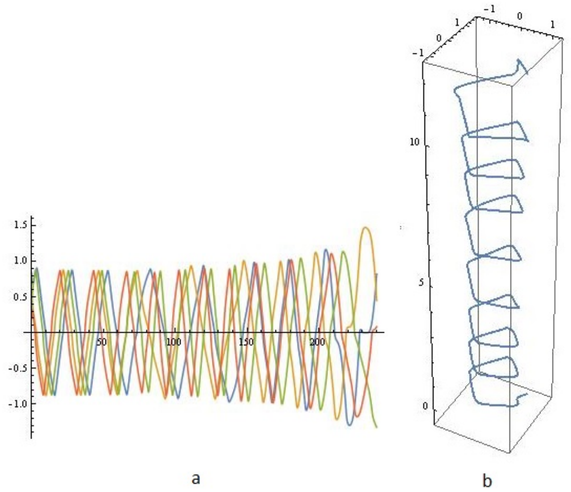


Figure 3. a) The solutions of differential system; (example II);
b) ParametricPlot3D[...]; (example II).

2. A modified adaptive Hopf–like oscillator with a suitable, user-fixed "*input function of coupling strength (IFCS)*"

Some simulations

Consider the following new modification of model (2)

$$\begin{cases} \dfrac{dx_i}{dt} &= (1-(|x_i|+|y_i|))sgn(x_i)+a_iy_i+c(t)(x_k-x_i) \\ & i,k\in\{1,2\},\ \ i\neq k \\ \dfrac{dy_i}{dt} &= (1-(|x_i|+|y_i|))sgn(y_i)-a_ix_i+c(t)(y_k-y_i) \\ & i,k\in\{1,2\},\ \ i\neq k \end{cases} \tag{3}$$

where

$$c(t)=\sum_{l=0}^{n}\alpha_l t^l$$

is a *input function of coupling strength (IFCS).*

Some simulations

II For given

$$a_1=0.2,\quad a_2=0.4,\quad c(t)=0.2-0.002t+0.000001t^2$$

the simulations on the system (3) for $x_1(0) = 0.9$; $x_2(0) = 0.13$, $y_1(0) = 0.8$, $y_2(0) = 0.12$ are depicted on Figure 3a.

Using operator $ParametricPlot3D[...]$ in CAS Mathematica for components of solution – $x_1(t), y_1(t)$ see Figure 3b.

II.1 For given

$$a_1 = 0.1, \quad a_2 = 0.04, \quad c(t) = 0.6 - 0.00082t + 0.00000004t^2$$

the simulations on the system (3) for $x_1(0) = 0.9$; $x_2(0) = 0.13$, $y_1(0) = 0.8$, $y_2(0) = 0.12$ are depicted on Figure 4a.

Using operator $ParametricPlot3D[...]$ in CAS Mathematica for components of solution – $x_1(t), y_1(t)$ see Figure 4b.

II.2 For given

$$a_1 = 0.07, \;\; a_2 = 0.01$$
$$c(t) = 0.15 - 0.0001t + 0.0000002t^2 - 0.000000001t^3$$

the simulations on the system (3) for $x_1(0) = 0.7$; $x_2(0) = 0.09$, $y_1(0) = 0.5$, $y_2(0) = 0.022$ are depicted on Figure 5a.

Using operator $ParametricPlot3D[...]$ in CAS Mathematica for components of solution – $x_1(t), y_1(t)$ see Figure 5b.

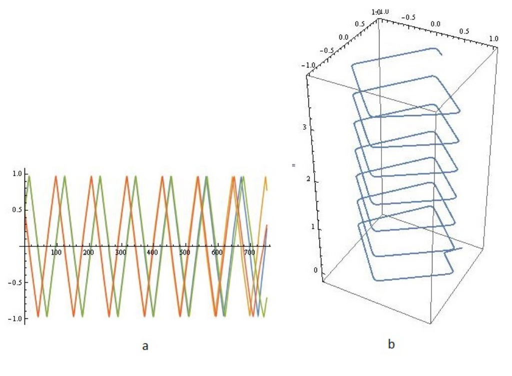


Figure 4. a) The solutions of differential system; (example II.1); b) ParametricPlot3D[...]; (example II.1).

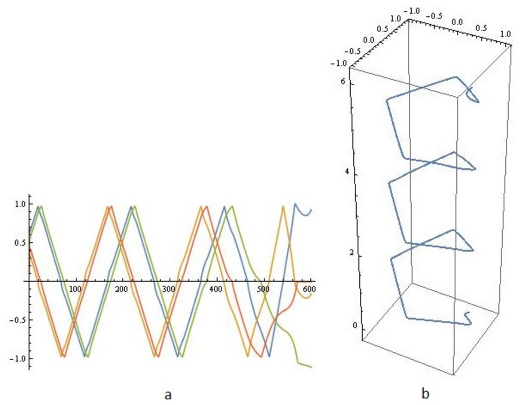


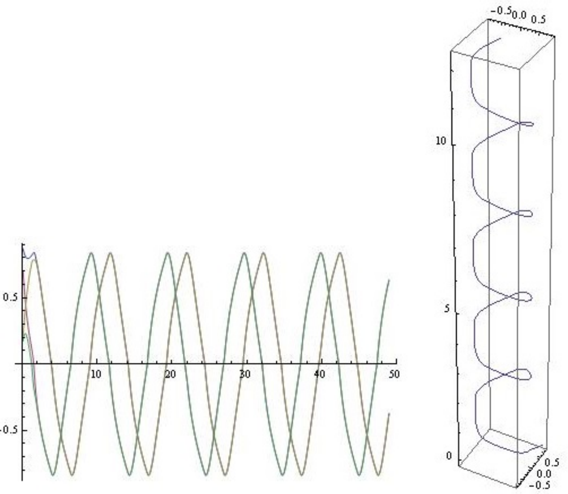
Figure 5. a) The solutions of differential system; (example II.2); b) ParametricPlot3D[...]; (example II.2).

II.3 For given

$$a_1 = 0.4, \quad a_2 = 0.9, \quad c(t) = -\frac{1}{32} + \frac{359}{720}t^2 - \frac{223}{448}t^4 + \frac{35}{128}t^6$$

the simulations on the system (3) for $x_1(0) = 0.9$; $x_2(0) = 0.13$, $y_1(0) = 0.8$, $y_2(0) = 0.12$ are depicted on Figure 6a.

Using operator $ParametricPlot3D[...]$ in CAS Mathematica for components of solution – $x_1(t), y_1(t)$ see Figure 6b.



a b

Figure 6. a) The solutions of differential system; (example II.3);
b) $ParametricPlot3D[...]$; (example II.3).

3. Concluding Remarks

In addition to the possibilities described in detail in [3]–[38], which our Web–application provides, in this paper we demonstrated some modules with which the planned Web–platform is upgraded, in the following directions:

- dynamics of the adaptive Hopf oscillator;
- dynamics of the new modified adaptive Hopf oscillator with a suitable, user-fixed "*input function of coupling strength (IFCS)*".

The following three examples using the new model (3) illustrate interesting dynamics typical of this type of oscillators. We hope that these results will be of interest to specialists working in this scientific direction.

Coupled nonlinear oscillators are abundant in biology, physics, and chemical reaction systems [40]–[42]. Some popular techniques for deriving the phase dynamics of coupled oscillators can be found in [43].

Defining the phase dynamics, however is not a trivial task. The literature provides an arsenal of solutions, but results are scattered and their formulation is far from standardized. The input function $c(t)$ can be considered as coupling function in networks of oscillators. We will not dwell on these important practical problems here.

We will explicitly note that the theoretical apparatus for studying the circuit implementation (design, fabricating, etc.) of the considered differential models is extremely complex and requires serious consideration before being adapted for its possible inclusion in our planned Web–based platform for scientific calculations. Of course, such an analysis is imperative and remains our top priority for future development.

Acknowledgment

The study of the first author is financed by the project No FP23-FMI-002 "Intelligent software tools and applications in research in Mathematics, Informatics, and Pedagogy of Education" of the Scientific Fund of the Paisii Hilendarski University of Plovdiv, Bulgaria.

The study of the second author was carried out with the financial support of the Fund "Scientific Research" of Plovdiv University Paisii Hilendarski, within the project: "A study of natural, demographic and cultural resources in the Western Rhodopes: potential for sustainable development of the region",contract No FP23-FS 003.

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