

# DOUBLE FUZZY NATURAL TRANSFORM TO SOLVE FUZZY CONVOLUTION VOLTERRA INTEGRAL EQUATIONS

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**Abstract.** *Integral equations have some applications in different fields, such as physics, biological models, and so on. They can be solved by different numerical or analytical methods. Specifically, fuzzy integral equations also appear in different problems, for example, optimal control theory, mathematical economics, and engineering. The study of fuzzy Volterra integral equations begins in Kaleva [2], Seikkala [5] and Mordeson and Newman [4], such integral equations being applied in control mathematical models.*

*Integral transforms constitute fundamental tools in operational calculus. They are mathematical operators that have been used widely in solving many practical problems in applied mathematics, physics and engineering. The precursor of the integral transforms is the Fourier transform, which is used to express functions in a finite interval. There are a number of works on the theories and applications of integral transforms, some of which are Laplace, Mellin and Hankel transforms. Subsequently, the concept of integral transforms was expanded to remove the necessity of finite intervals.*

*The Natural transform evolved from the Fourier integral and it converges into Laplace transform and Sumudu transform given a unit value to each transform parameters respectively. The definition of the Natural transform and the study of their properties and applications were first done by Khan and Khan [3]. Several research works in connection with Natural transforms properties and applications are published in [1].*

*The subject of this paper is to apply the double fuzzy Natural transform for solving convolution type Volterra fuzzy integral equation in two variables*

$$u(s, t) = g(s, t) \oplus (FR) \int_0^s k_1(s-x, t) \odot u(x, t) dx \oplus (FR) \int_0^t k_2(s, t-y) \odot u(s, y) dy,$$

*where  $k_1, k_2 : [0, b] \times [0, d] \rightarrow R$  are continuous functions and  $g, u : [0, b] \times [0, d] \rightarrow E^1$  are continuous fuzzy-number valued functions.*

*We give some preliminaries on fuzzy numbers, fuzzy functions and fuzzy integrals. Then we provide the proposed double fuzzy Natural transform and*

*its convolution theorem. Later we construct a detailed procedure for solving two-dimensional fuzzy convolution Volterra integral equation using double fuzzy Natural transform. Numerical example is provided to demonstrate the proposed method.*

**Key words:** Double fuzzy Natural transform, Fuzzy Volterra integral equation.

**Mathematics Subject Classification:** 44A30, 45A05, 45D05.

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